AP Calculus

NO CALCULATORS!

If $y = \cos^2(2x)$, find $\dfrac{dy}{dx}$.

$$-4\sin(2x)\cos(2x)$$

Find the slope of the line tangent to the graph of $y = \ln\sqrt{x}$ at $\left(e^2, 1\right)$.

$$\frac{1}{2e^2}$$

The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between (1, 1) and (9, 3). What are the coordinates of this point?

$$(4, 2)$$

If $f(x)=\frac{x^4}{2}-\frac{x^5}{10}$, the derivative of f attains its maximum at what value of x?

$$x = 3$$

Evaluate

$$\lim_{x \to 0} \left(\frac{e^{3x} - 1}{\sin 2x} \right)$$

$$\frac{3}{2}$$

Given $f(x) = \begin{cases} \dfrac{x^2 - 2x + 1}{x - 1} & \text{for } x \neq 1 \\ k & \text{for } x = 1 \end{cases}$

Determine the value of k for which f is continuous for all real x.

0

Find the average value of $f(x) = e^{2x} + 1$ on the interval $0 \leq x \leq \frac{1}{2}$.

e

A point moves on the x-axis in such a way that its velocity at any time $t > 0$ is given by $v(t) = \dfrac{e^t}{t}$. At what value of t does v attain its minimum?

$$t = 1$$

$$\int \frac{4x}{1+x^2}\,dx$$

$$2\ln\left(1+x^2\right)+C$$

Let $f(x) = x^4 + ax^2 + b$. The graph of f has a relative maximum at (0, 1) and an inflection point when $x = 1$. Find the values of a and b.

$$a = -6$$

$$b = 1$$

$$\int_1^2 \frac{x^2 - x}{x^3}\,dx$$

$$\ln 2 - \frac{1}{2}$$

The edge of a cube is increasing at the uniform rate of 0.2 inches per second. At the instant when the total surface area becomes 150 square inches, what is the rate of increase, in cubic inches per second, of the volume of the cube?

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$$\int_0^{\sqrt{3}} \frac{x\,dx}{\sqrt{1+x^2}}$$

1

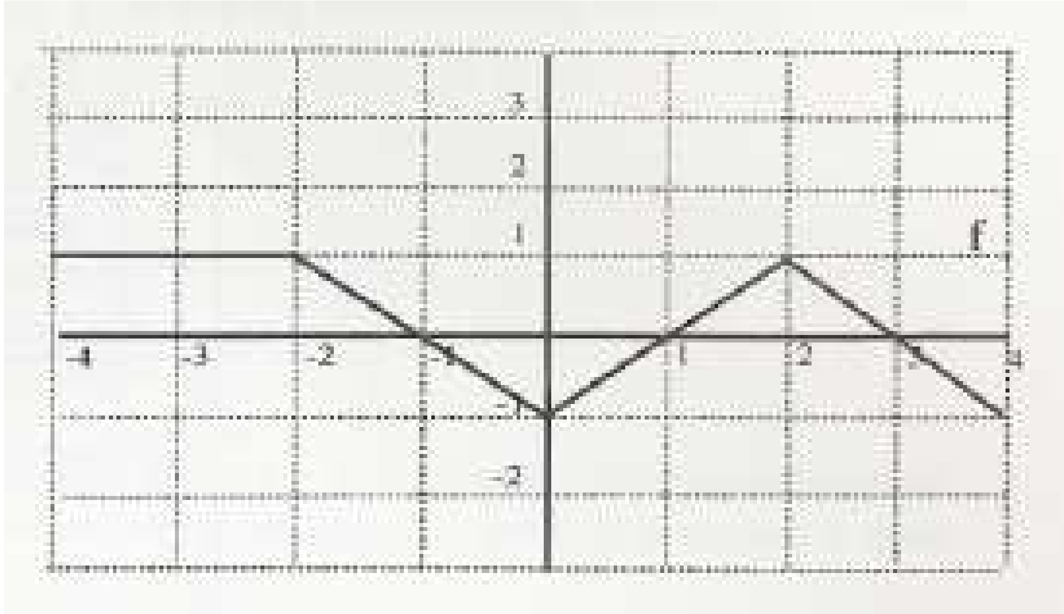
$$\int x\left(x^2-1\right)^4 dx$$

$$\frac{1}{10}\left(x^2-1\right)^5+C$$

If $y = e^{kx}$ find $\dfrac{d^5 y}{dx^5}$.

(k is a constant.)

$$k^5 e^{kx}$$



The graph of f is shown above. Which of the following statements are true?

I. $f(2) > f'(1)$

II. $\int_0^1 f(x)\,dx > f'(3.5)$

III. $\int_{-1}^1 f(x)\,dx > \int_{-1}^2 f(x)\,dx$

II only

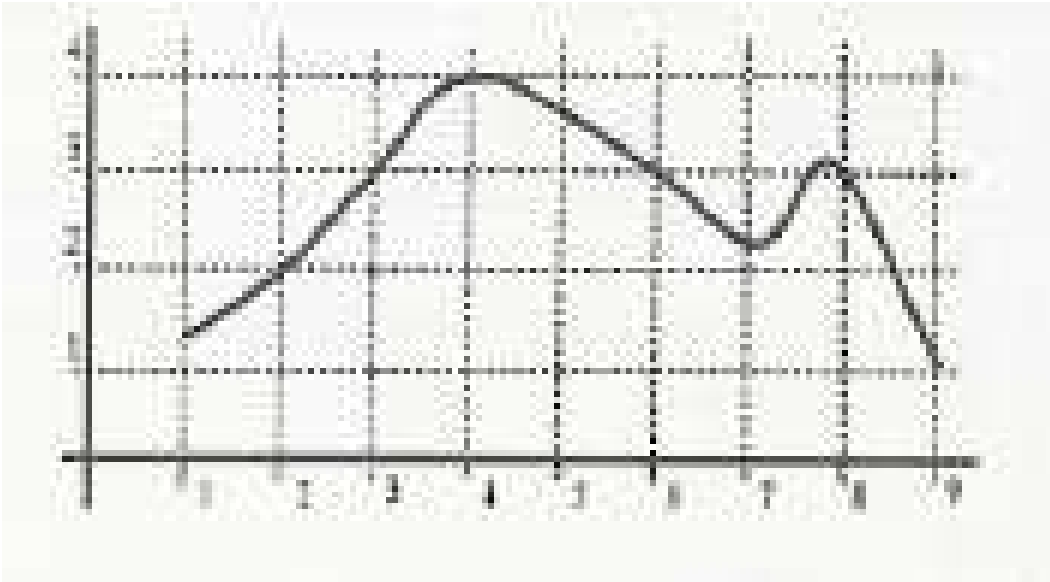
Find the slope of the tangent to the curve $xy - 2y + 4y^2 = 6$ at the point where $y = 1$.

$$-\frac{1}{10}$$

Get out your CALCULATORS!

It's calculator-throwing time!


TEXAS INSTRUMENTS
TI-83
PLEASE DO NOT
THROW ME!
STAT PLOT TBLSET FORMAT CALC TABLE
Y= WINDOW ZOOM TRACE GRAPH
QUIT INS
2nd MODE DEL
A-LOCK LINK LIST
ALPHA X,T,θ,n STAT
TEST ANGLE DRAW DISTR
MATH MATRX PRGM VARS CLEAR
SIN COS TAN
LOG 7 8 9
LN 4 5 6
RCL MEM
STO▸ 1 2 3 +
OFF CATALOG ANS ENTRY SOLVE
ON 0 . (−) ENTER



The graph of f over the interval [1, 9] is shown above. Find a midpoint approximation with 4 equal subdivisions for $\int_1^9 f(x)\,dx$.

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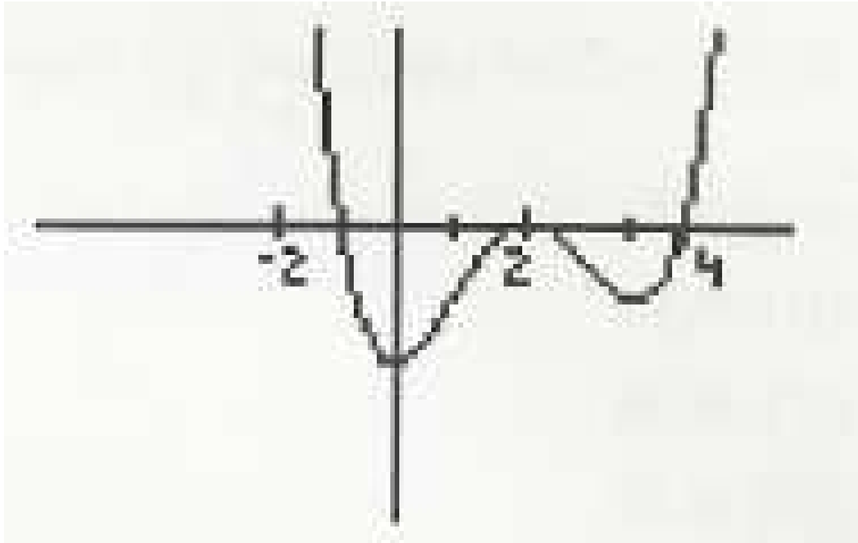
If $f(x) = 2x + \sin x$ and the function g is the inverse of f, find $g'(2)$.

$$
\begin{aligned}
g'(2) &= \frac{1}{f'(0.68403666)} \\
&= \frac{1}{2+\cos(0.68403666)} \\
&= \boxed{0.360}
\end{aligned}
$$



This a graph of f', <u>not</u> f.

Let f be a function that has domain $[-2,5]$. The graph of f' is shown above.

Which of the following statements are TRUE?

I. f has a relative maximum at $x=-1$.

II. f has an absolute minimum at $x=0$.

III. f is concave down for $-2<x<0$.

IV. f has inflection points at $x=0$, $x=2$, $x=3$.

I, III, and IV

On which interval is the graph of $f(x) = 4x^{3/2} - 3x^2$ both concave down and increasing?

(A) $(0, 1)$

(B) $\left(0, \dfrac{1}{2}\right)$

(C) $\left(0, \dfrac{1}{4}\right)$

(D) $\left(\dfrac{1}{4}, \dfrac{1}{2}\right)$

(E) $\left(\dfrac{1}{4}, 1\right)$

(E)

If $f(x) > 0$ for all x, solve the differential equation $$\frac{dy}{dx} = 3\sqrt{xy}.$$

Initial condition: $f(0) = 1$.

Write the answer in the form $y = f(x)$.

$$y = \left(x^{3/2} + 1 \right)^2$$

If $\sin(3x)-1=\int_a^x f(t)\,dt,$

find the value of a.

<u>Hint</u>: The first step is "differentiate both sides" of the equation. Use the <u>Fundamental Theorem of Calculus</u> on the right side.

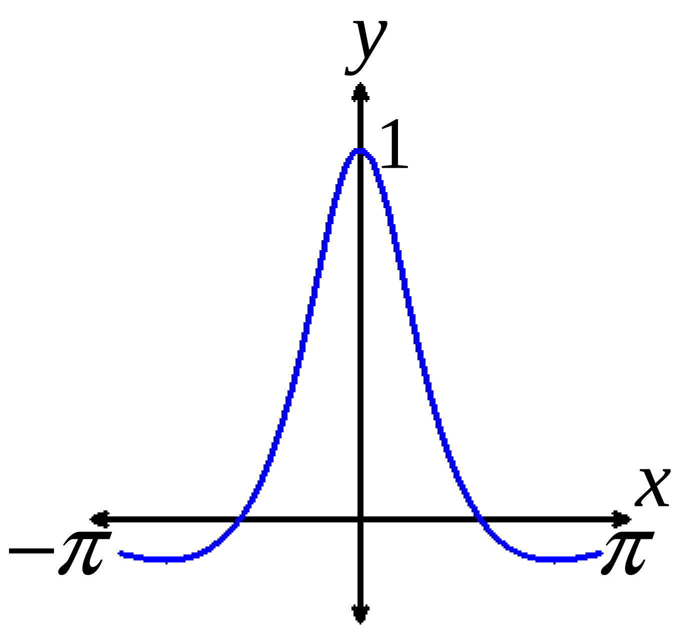
$$\frac{\pi}{6} \approx 0.524$$

If $xy^2 = 20$ and x is decreasing at the rate of 3 units per second, find the rate at which y is changing when $y = 2$.

$$\frac{3}{5}$$

If $f(x)$ is defined on $-\pi \leq x \leq \pi$ and $\dfrac{dy}{dx} = \dfrac{\cos x}{x^2+1}$, determine <u>how many</u> relative extrema and <u>how many</u> inflection points the graph of $y = f(x)$ has.

$y = f(x)$ has <u>two</u> relative extrema and <u>three</u> inflection points.

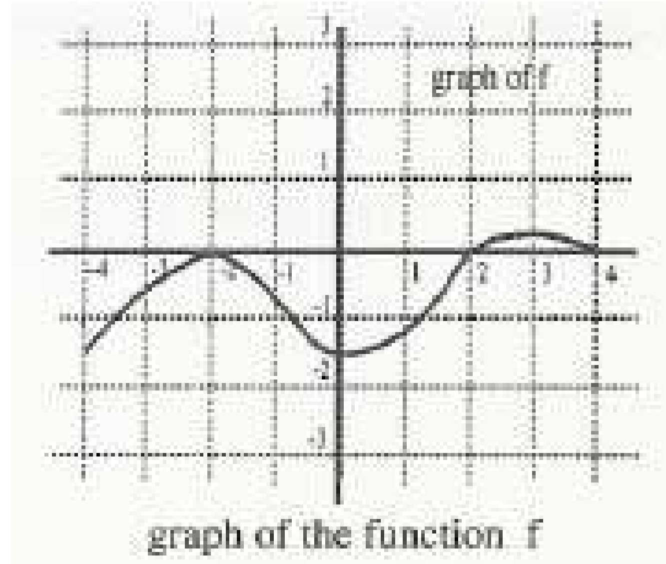


This is the graph of f'.

Since f' has two zeros in the interval $[-\pi, \pi]$, f has two extrema.

Since f' has three extrema in the interval $[-\pi, \pi]$, f has three inflection points.

The graph of the function f is shown at the right. If the function G is defined by $G(x) = \int_{-4}^{x} f(t)\,dt$, for $-4 \le x \le 4$, which of the following statements about G are true?



I. G is increasing on (1, 2).

II. G is decreasing on $(-4, -3)$.

III. $G(0) < 0$

II and III